Rheological behavior of dense assemblies of granular materials

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Project objectives

I. Develop validated continuum models for frictional flow of granular materials in the quasi-static and intermediate regime, including regime transitions from
   - A. Quasi-static to intermediate
   - B. intermediate to inertial

II. Develop closure models in terms of particle properties

Synopsis of first year activity

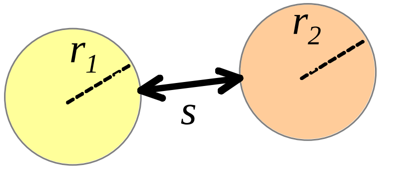
- Simulated shear flow in periodic domains with constant volume or constant normal stress conditions using discrete element method (DEM); assessed the available hypoplastic models (Princeton)

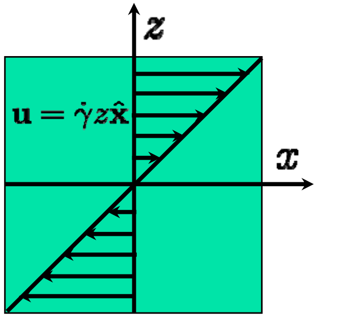
- Developed instruments and collected in-situ data from stress sensors on a shearing surface in Jenike cell and axial-flow Couette devices. (CCNY)

- Simulated shear flow in wall-bounded domains with constant volume or constant normal stress conditions (ISU)

Simulation methodology

- Discrete element method
  - linear spring-dashpot model with spring stiffness k
  - inter-particle friction coefficient $\mu = 0.1$
  - restitution coefficient $e = 0.7$





- Homogeneous simple shear
  - 3D zero-gravity system using periodic domain
  - with Lees-Edwards boundary condition

- Cohesion force modeled as van der Waals force*

$$F_{vdW} \approx \frac{Ad^6}{6s^2(s+2d)^2(s+d)^3} \xrightarrow{s<<d} \frac{Ad}{24s^2}$$

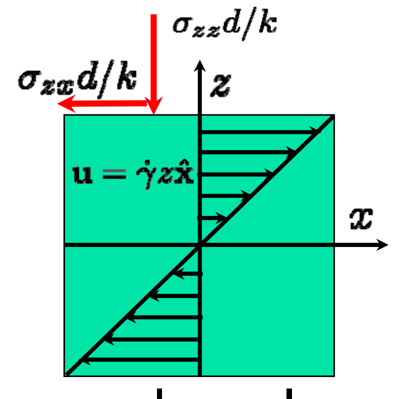
$$s_{\min} = 4 \times 10^{-5} d$$
$$s_{\max} = 0.25d$$

* Seville et al, Powder Technology 113, 261 (2000).

Steady shear simulations

- Computational system
  - number of particles $N = 2000$
  - system volume V constant for constant volume simulation
  - scaled stiffness $k^* = k/\rho d^3 \dot{\gamma}^2$
  - cohesion strength $Bo^* = F_{\mathrm{vdW}}^{\max}/kd \approx A/24ks_{\min}^2$

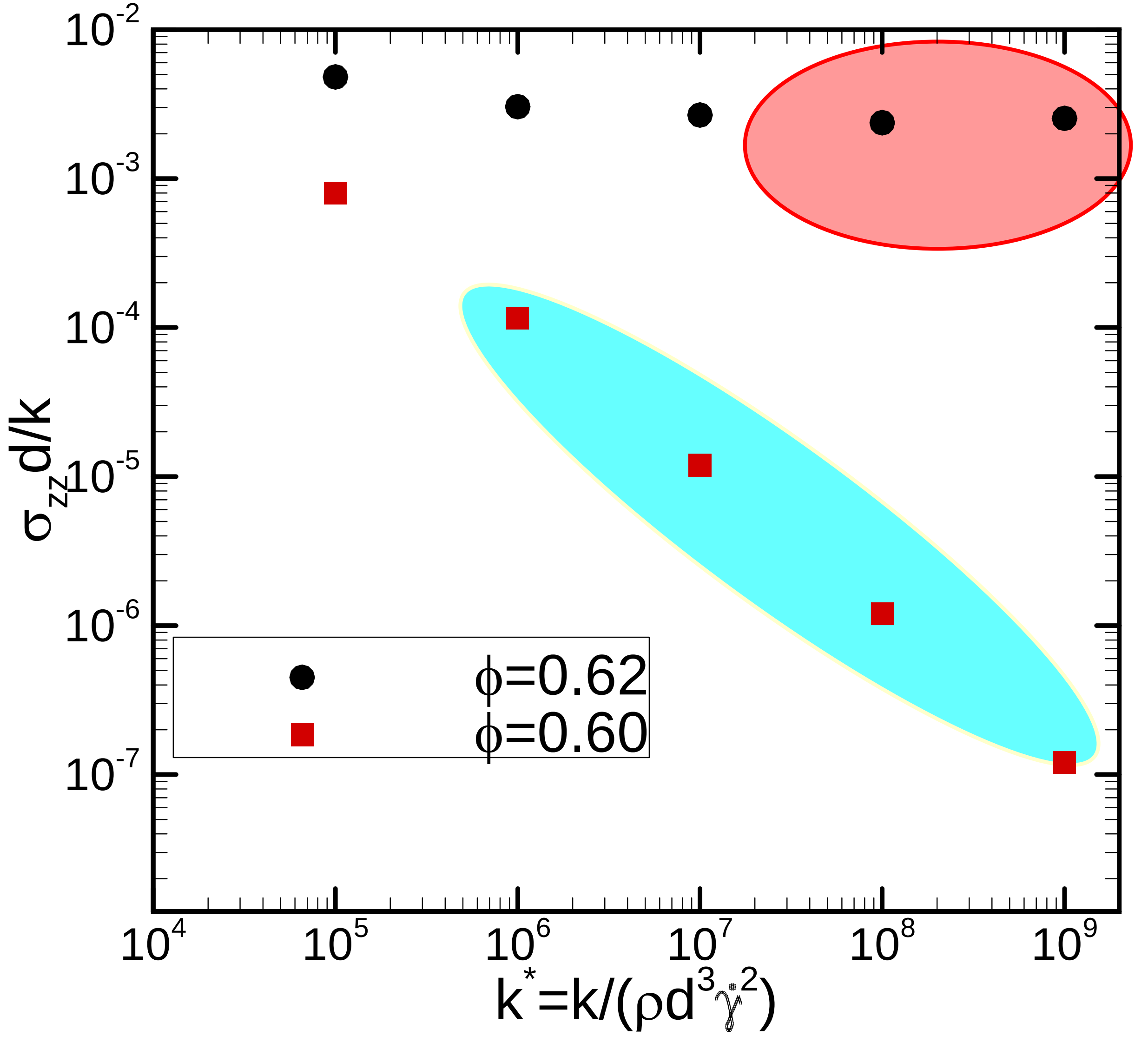
- Continuum fields
  - solid volume fraction ϕ
  - scaled stress $\sigma_{ij} d/k$
  - apparent friction coefficient $\mu_{\mathrm{app}} = \left| \dfrac{\sigma_{zx}}{\sigma_{zz}} \right|$



$\sigma_{zz} d/k$ constant for constant normal stress

Flow regimes

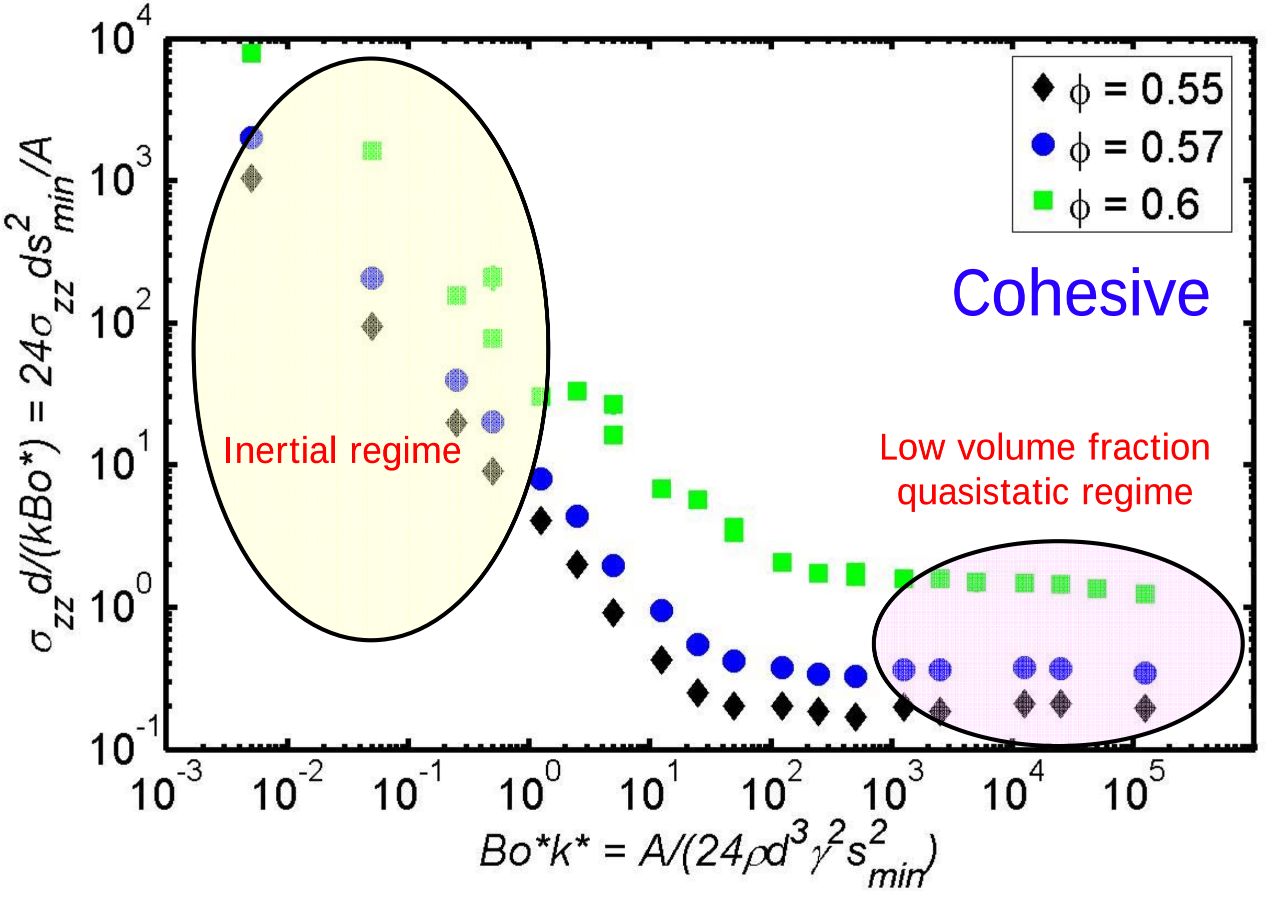
Non-cohesive



High volume fraction quasistatic regime

Inertial regime

Flow regimes

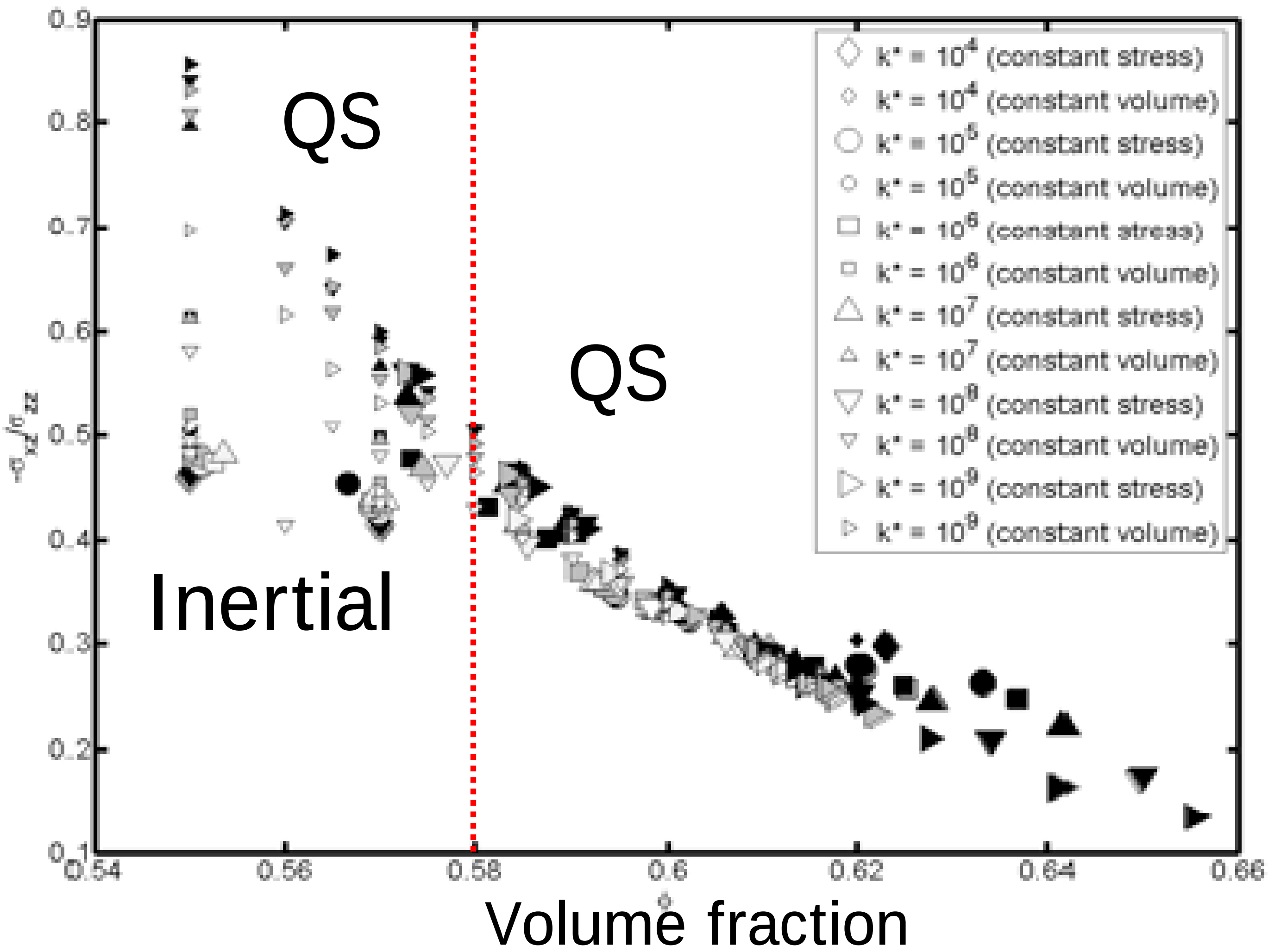


Both the stress and shear rate
are now scaled differently

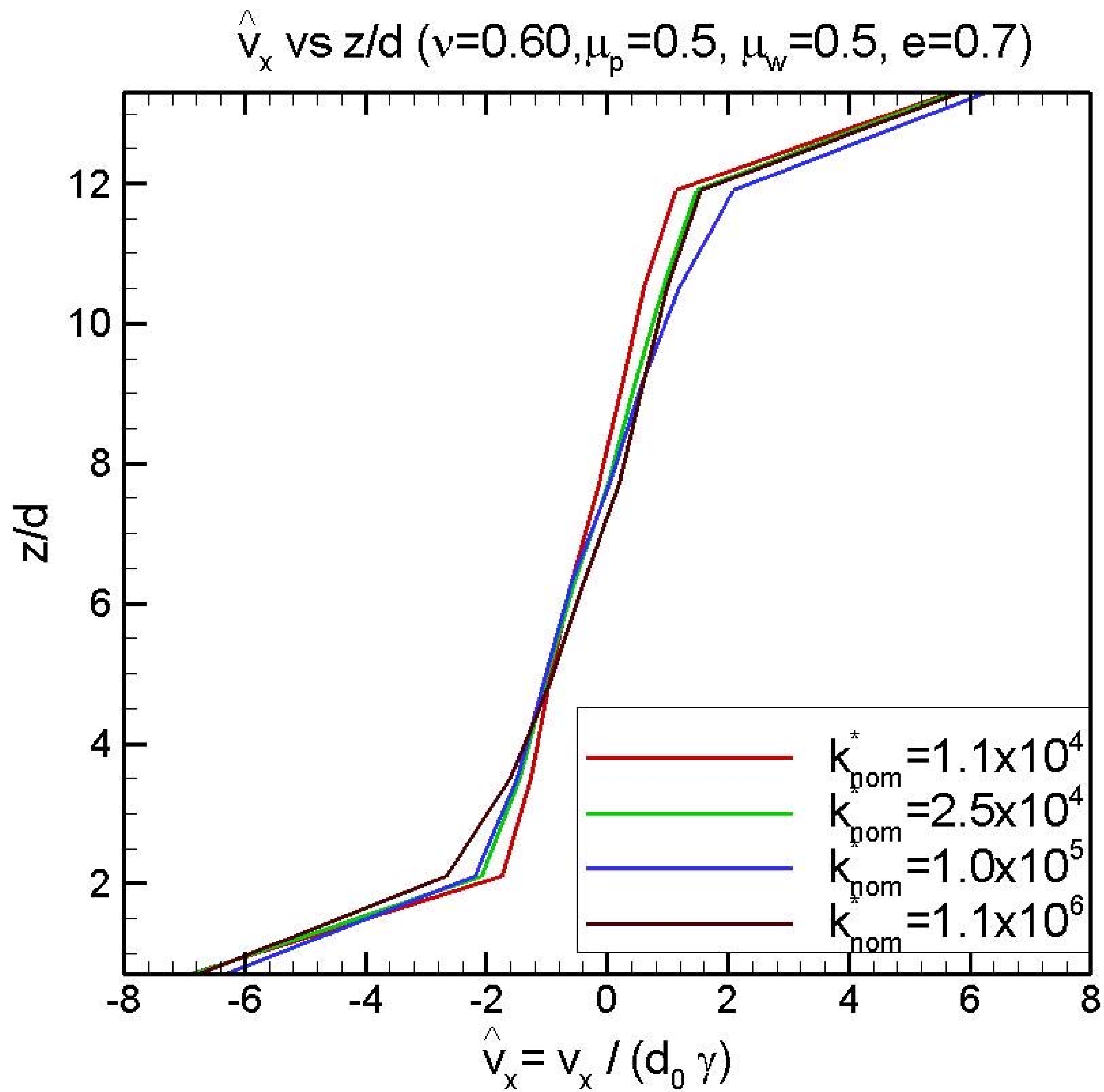
Apparent friction coefficient

$$\mu_{app} = \left|\frac{\sigma_{zx}}{\sigma_{zz}}\right|$$

In the QS regime, apparent friction coefficient is a strong, but simple, function of volume fraction



Simulation results with wall

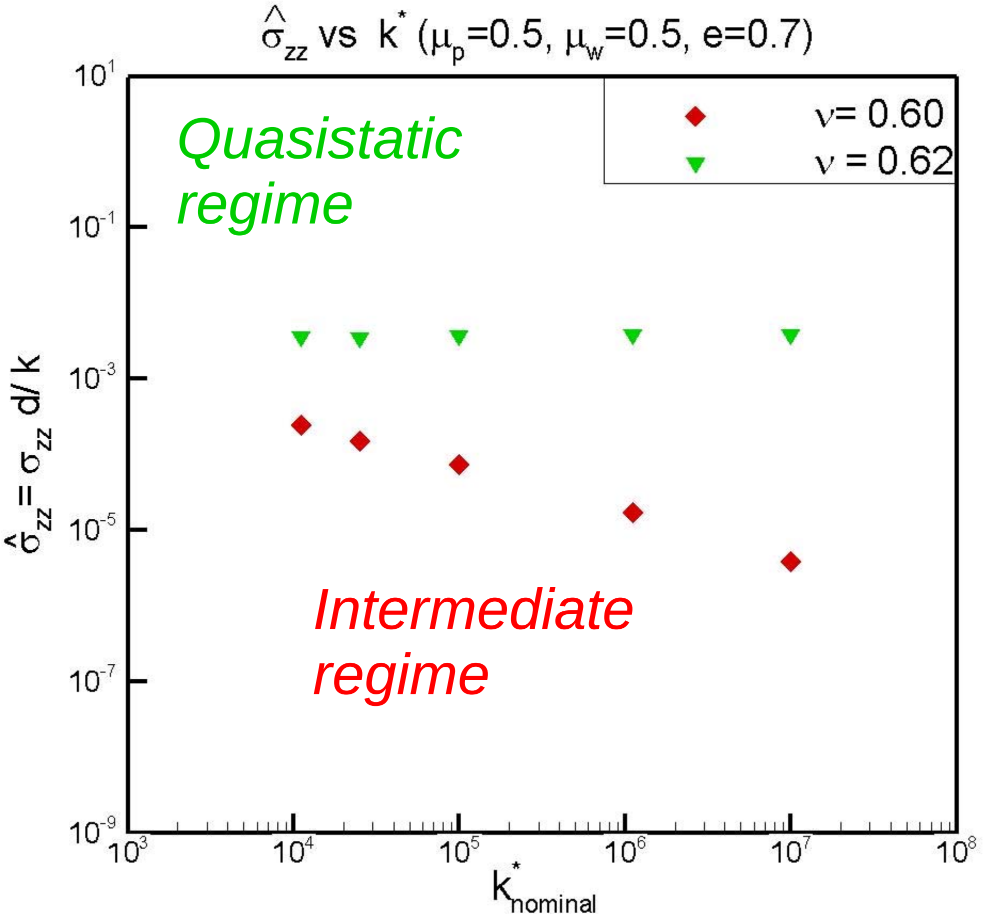


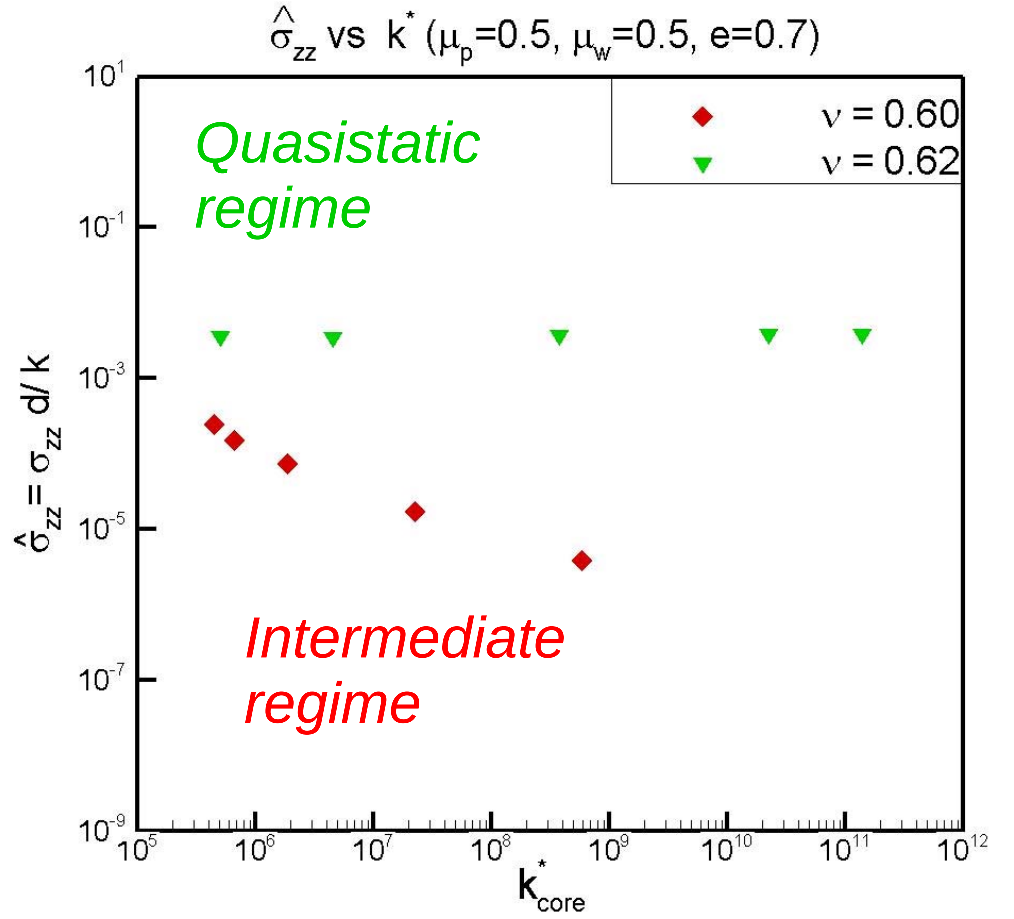
Non-cohesive

$$k^*_{nom} = k / \rho d^3 \dot{\gamma}^2_{nom}$$

$$k^*_{core} = k / \rho d^3 \dot{\gamma}^2_{core}$$

Simulation results with wall





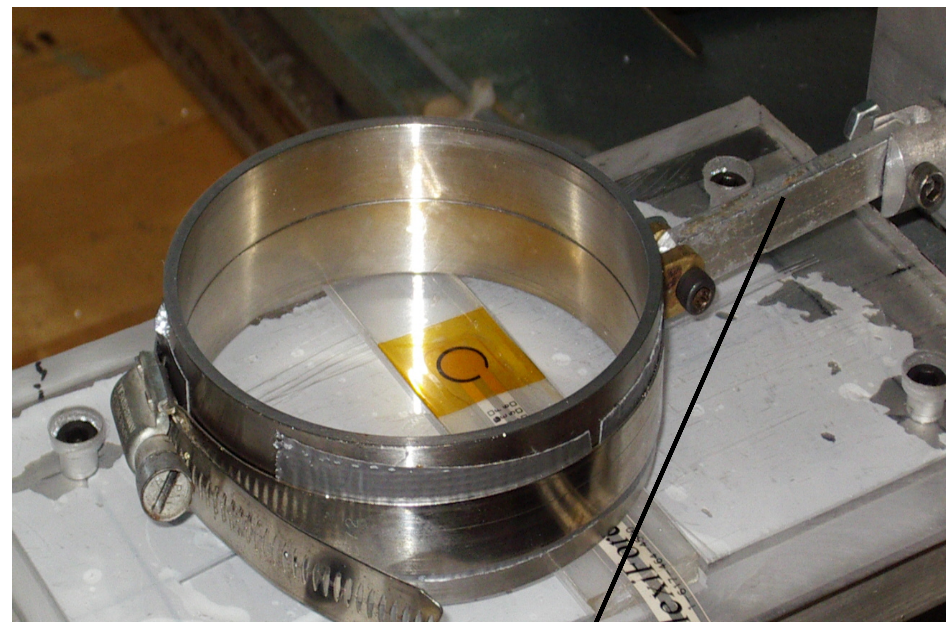
$k^*_{nom} = k / \rho d^3 \dot{\gamma}^2_{nom}$ Non-cohesive $k^*_{core} = k / \rho d^3 \dot{\gamma}^2_{core}$

Jenike Cell experimental setup

Ring Cell

Stress Sensor

Detail



Activating Arm

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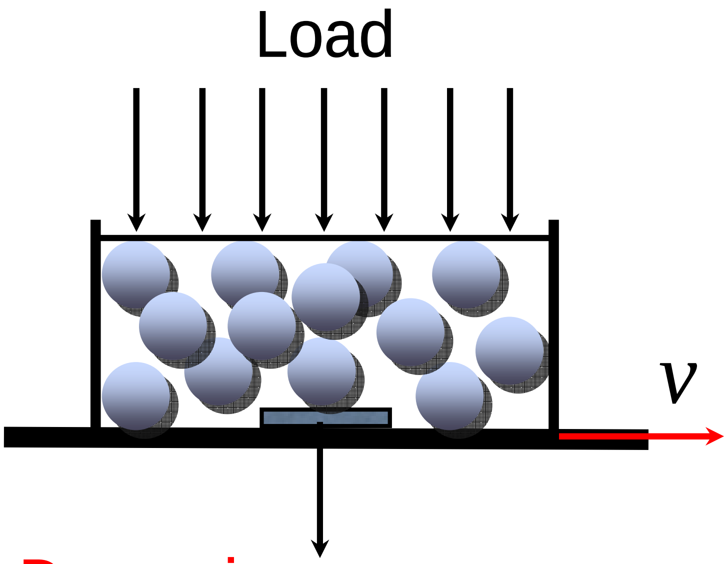
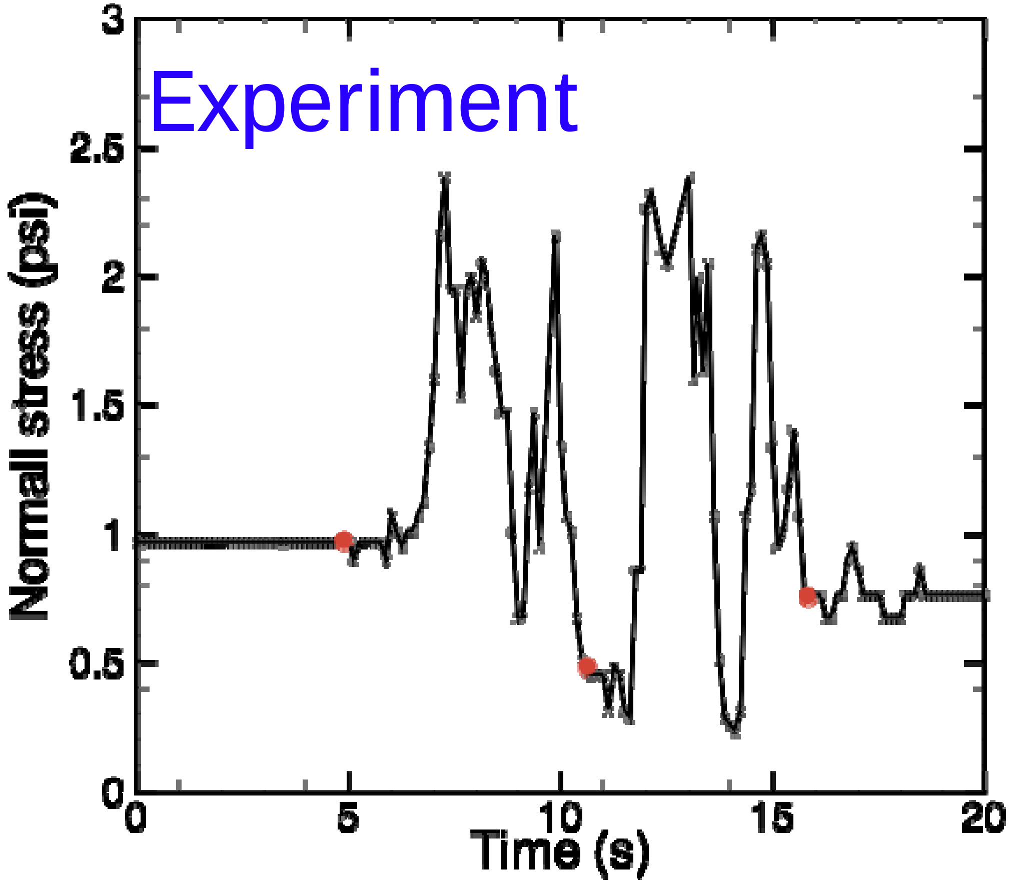
Jenike cell simulations

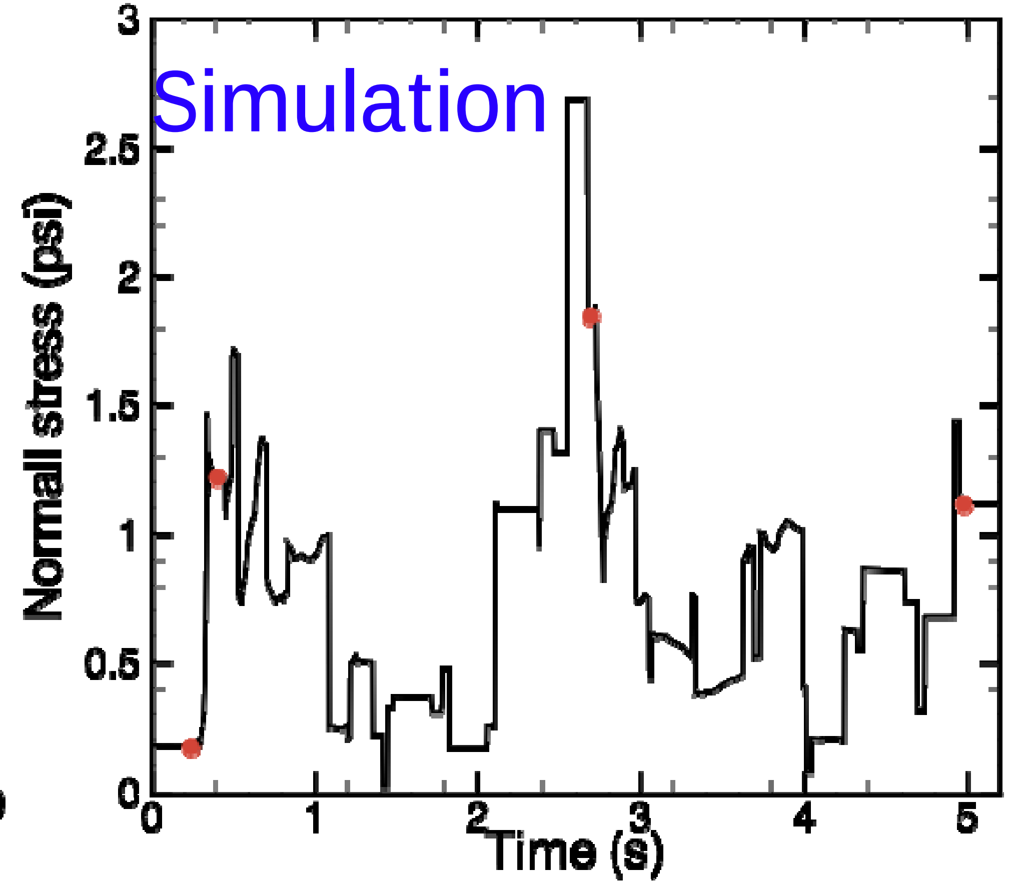
- Simulations set up as closely to the CCNY experiment as possible
- Stresses computed by dividing the sum of the contact forces acting on the wall by the area of wall or sensor
- Dynamic sensor mimics the experimental sensor; static sensors do not move relatively to the particles
- Case: external load 1psi; velocity 16 mm/sec



- Dynamic sensor move
- Static sensor stay

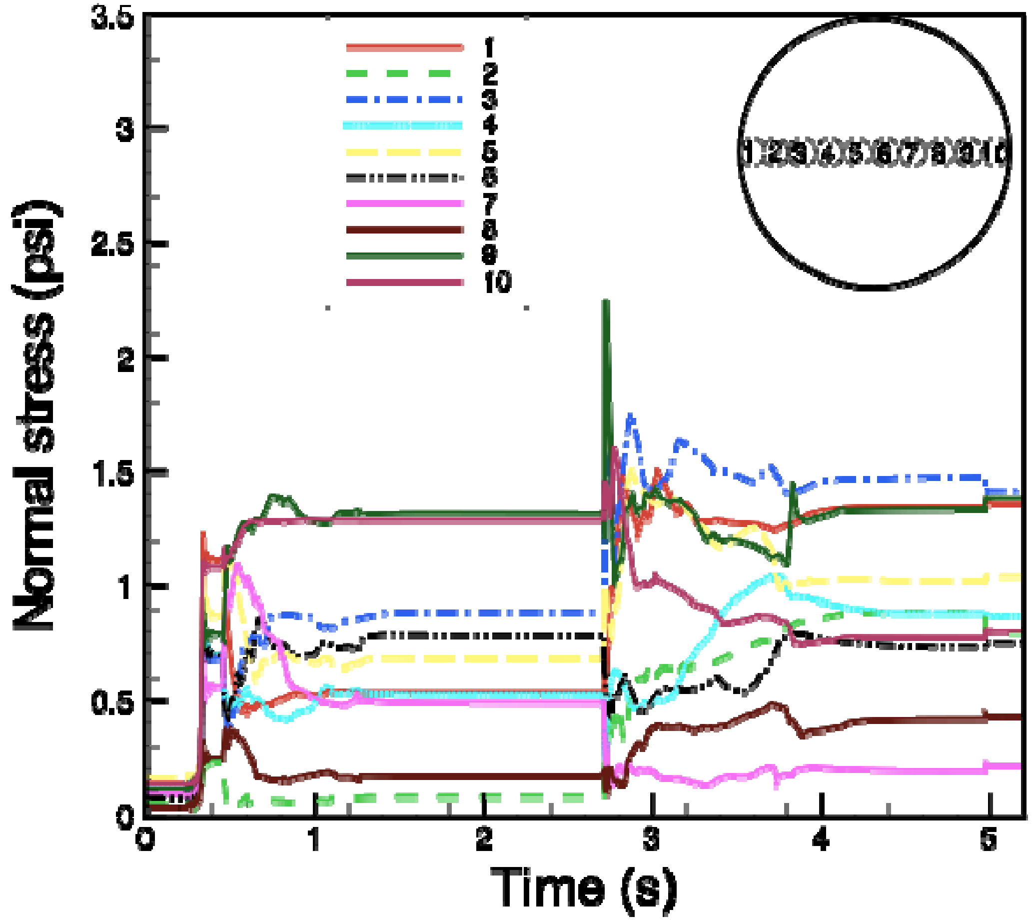
Stress on dynamic sensor





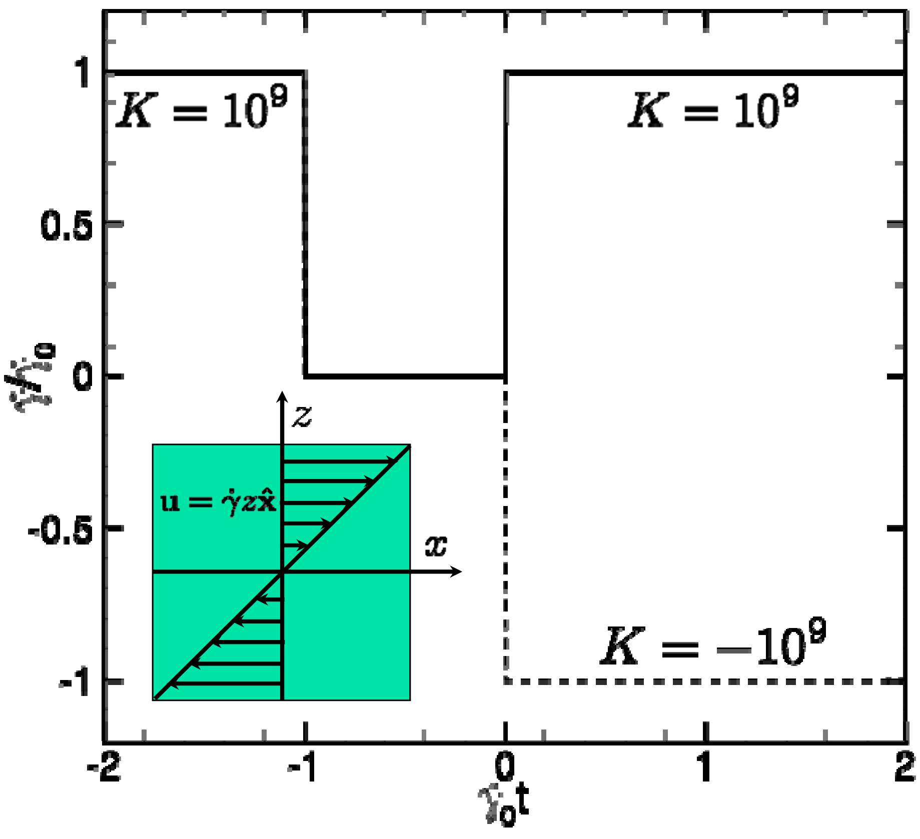
- Stress fluctuates significantly around one psi
- Fluctuating range agrees

Stress on static sensors



- Stress is spatially inhomogeneous; temporally steady after short time
- Fluctuation on dynamic sensor is largely due to spatial inhomogeneity and finite sensor size

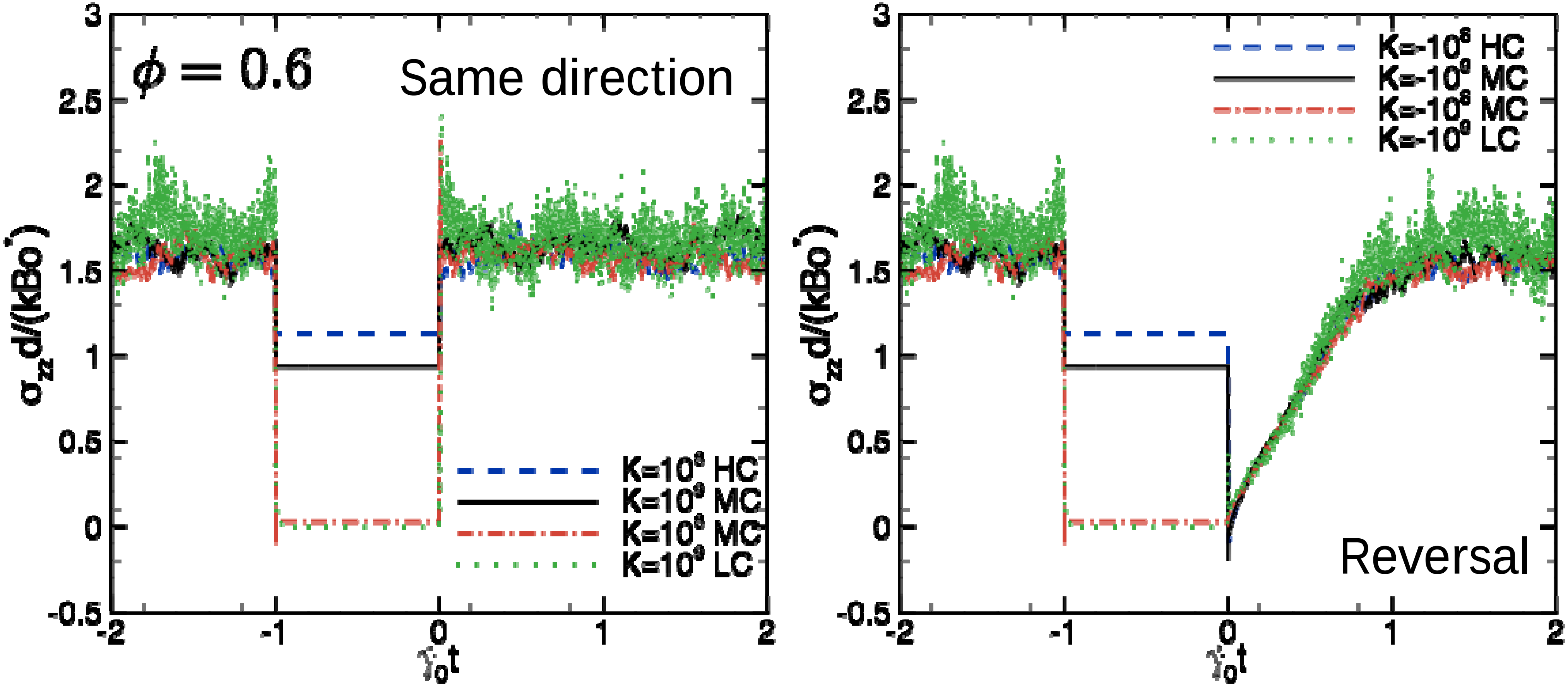
Unsteady shear simulations



Stop-and-go shear

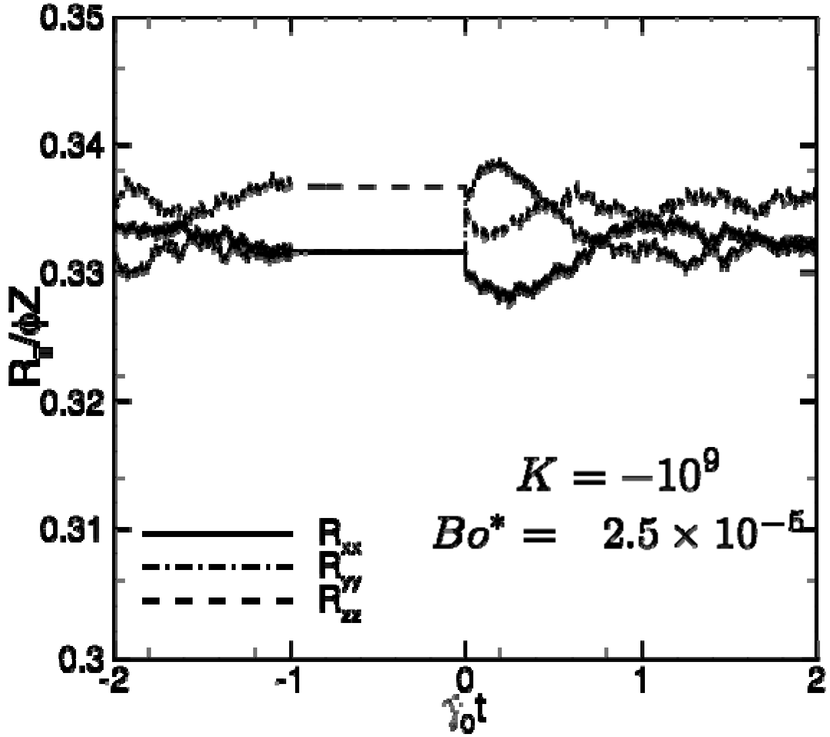
- System sheared to steady state at $\dot{\gamma}_0$
- Shear stops for $\dot{\gamma}_0 t = 1$
- Shear resumes in the same direction or in reversed direction $K = \mathrm{Sign}(\dot{\gamma})k^*$
- Cohesion strength
  - HC $Bo^* = 1.25 \times 10^{-4}$
  - MC $Bo^* = 2.5 \times 10^{-5}$
  - LC $Bo^* = 5 \times 10^{-7}$

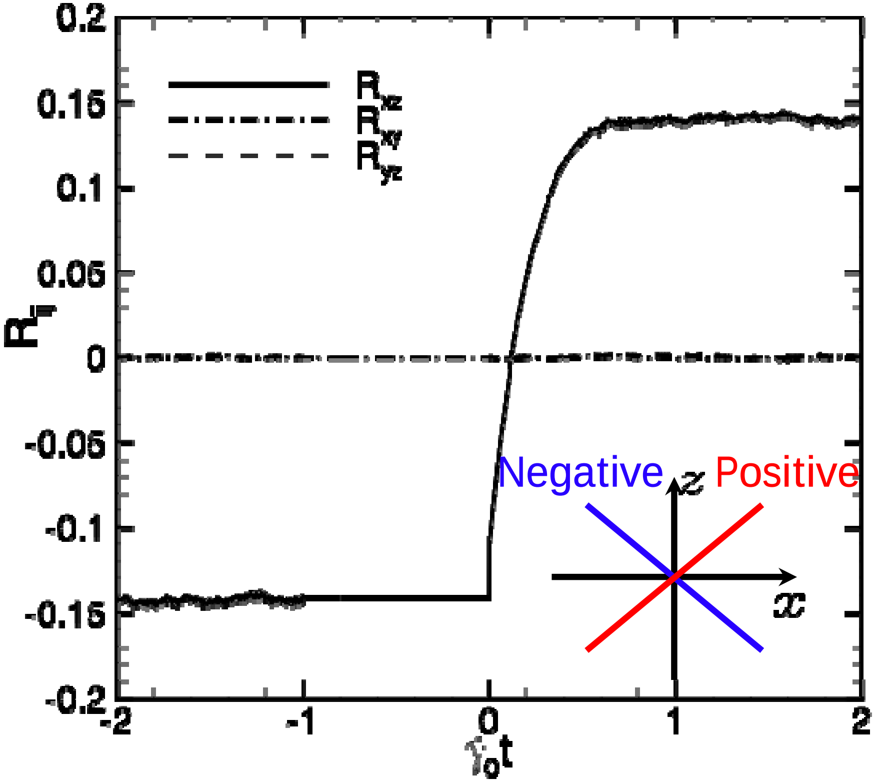
Normal stress evolution



- Independent of shear rate and cohesion strength
- Transition after shear reversal need strain of order unity to recover
- Stress relaxation depends on how “deep” in the quasistatic regime

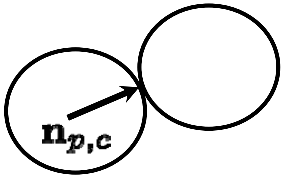
Characterize microstructure





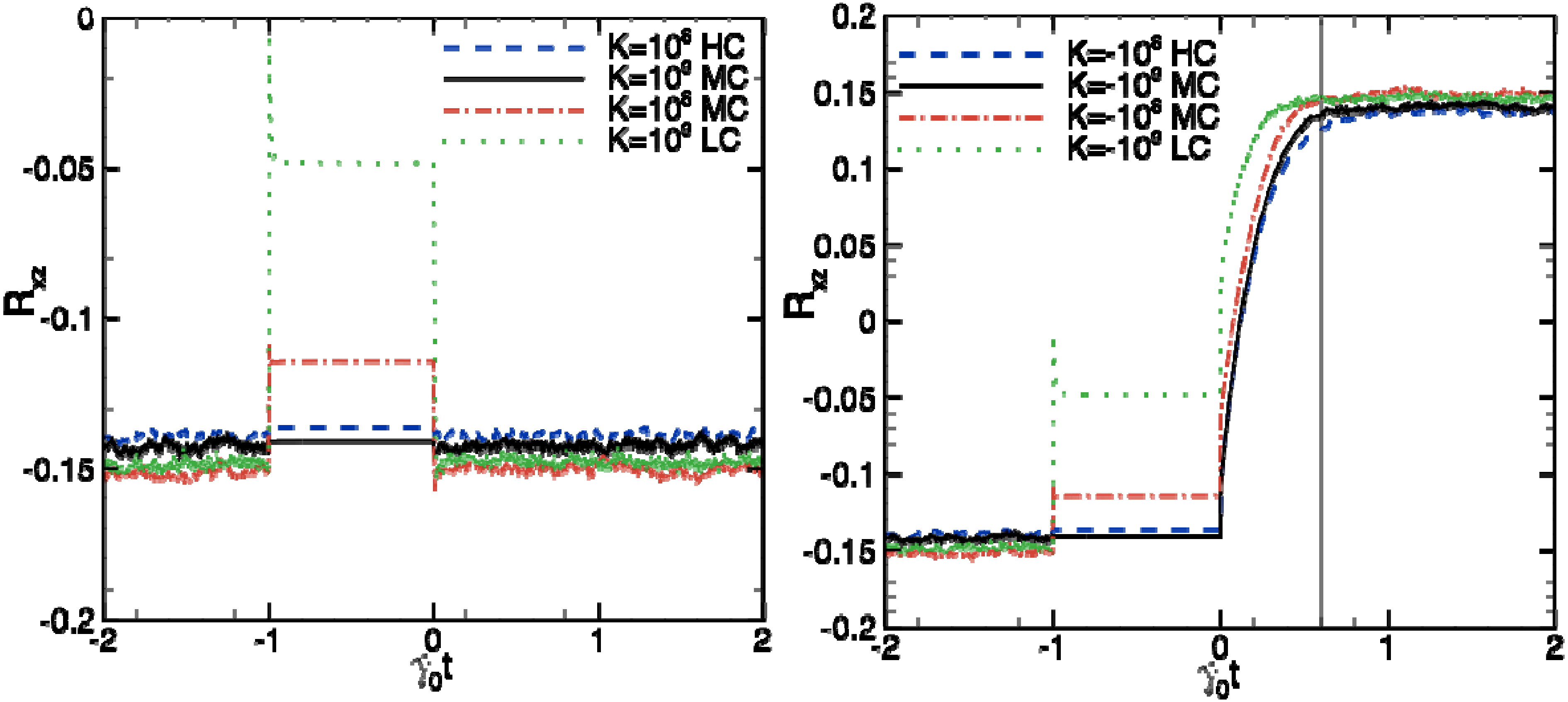
- Fabric tensor: average of outer product of particle contact normals

$$\mathrm{R} = \langle \mathbf{n}_{p,c}\mathbf{n}_{p,c} \rangle = \frac{\phi}{N}\sum_{p=1}^{N}\sum_{c=1}^{c_p}\mathbf{n}_{p,c}\mathbf{n}_{p,c}$$



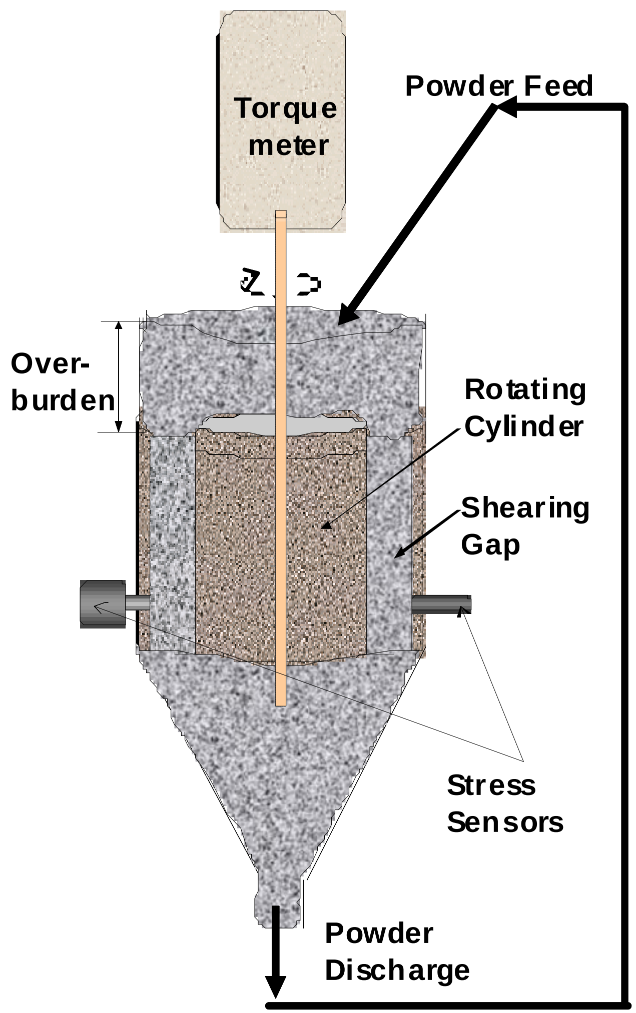
- R_{xz} magnitude indicates the microstructure anisotropy strength; sign indicates the anisotropy direction

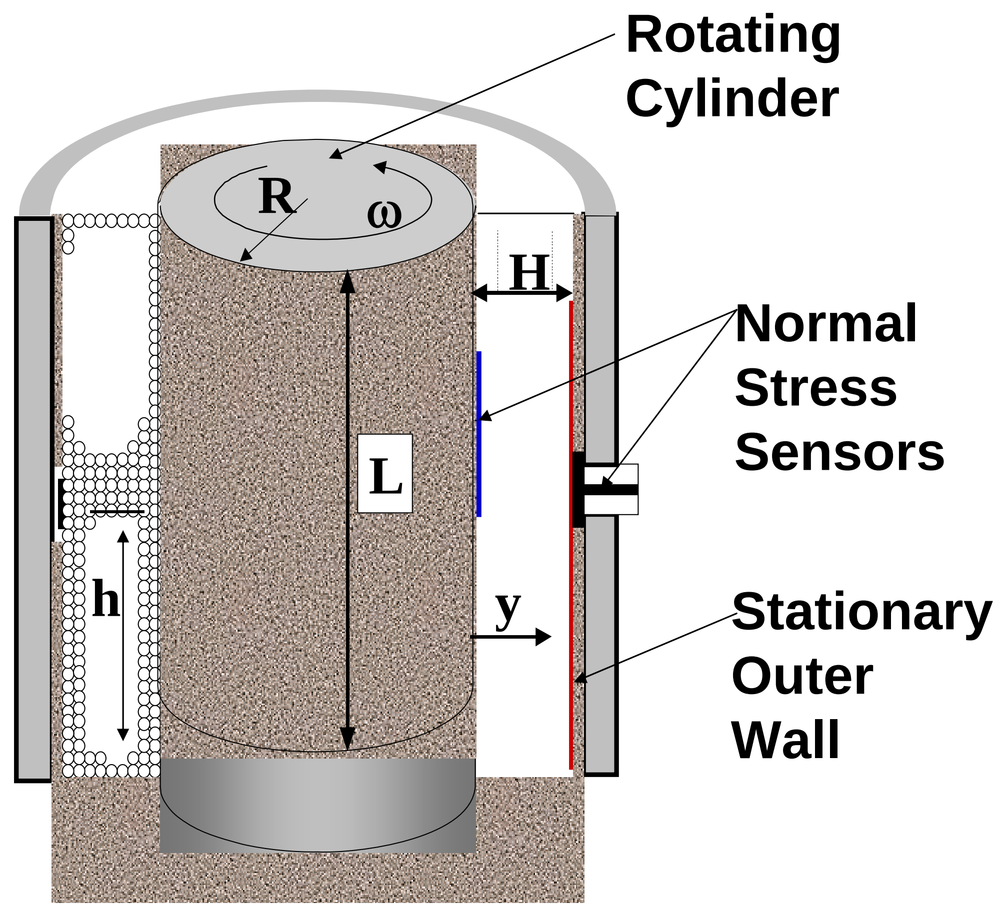
Anisotropy evolution



- Microstructure evolves in a correlated way with stress
- Transition after reversal is gradual and requires comparable strain to reach steady state

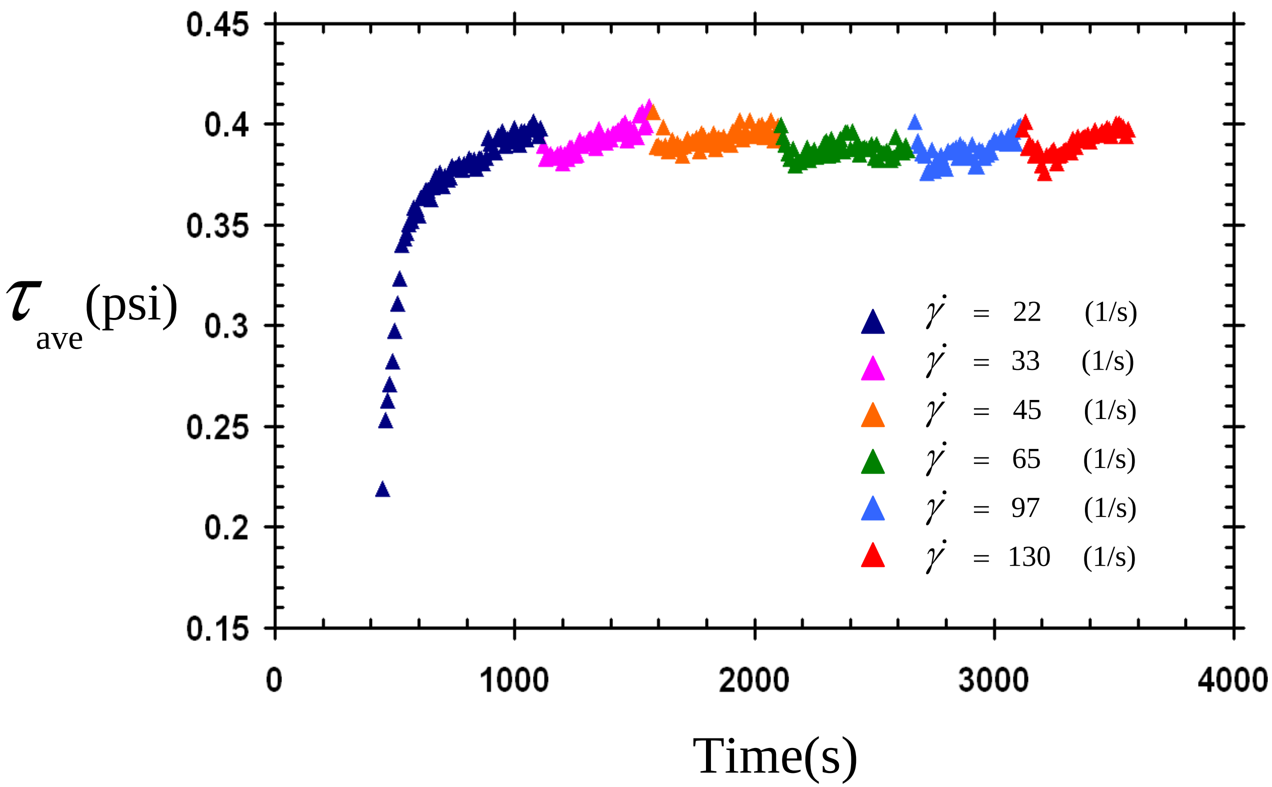
Axial Flow Couette Apparatus





- Allows closed-bed (Batch) and open-bed (continuous) experiments

Flow regimes in a batch Couette

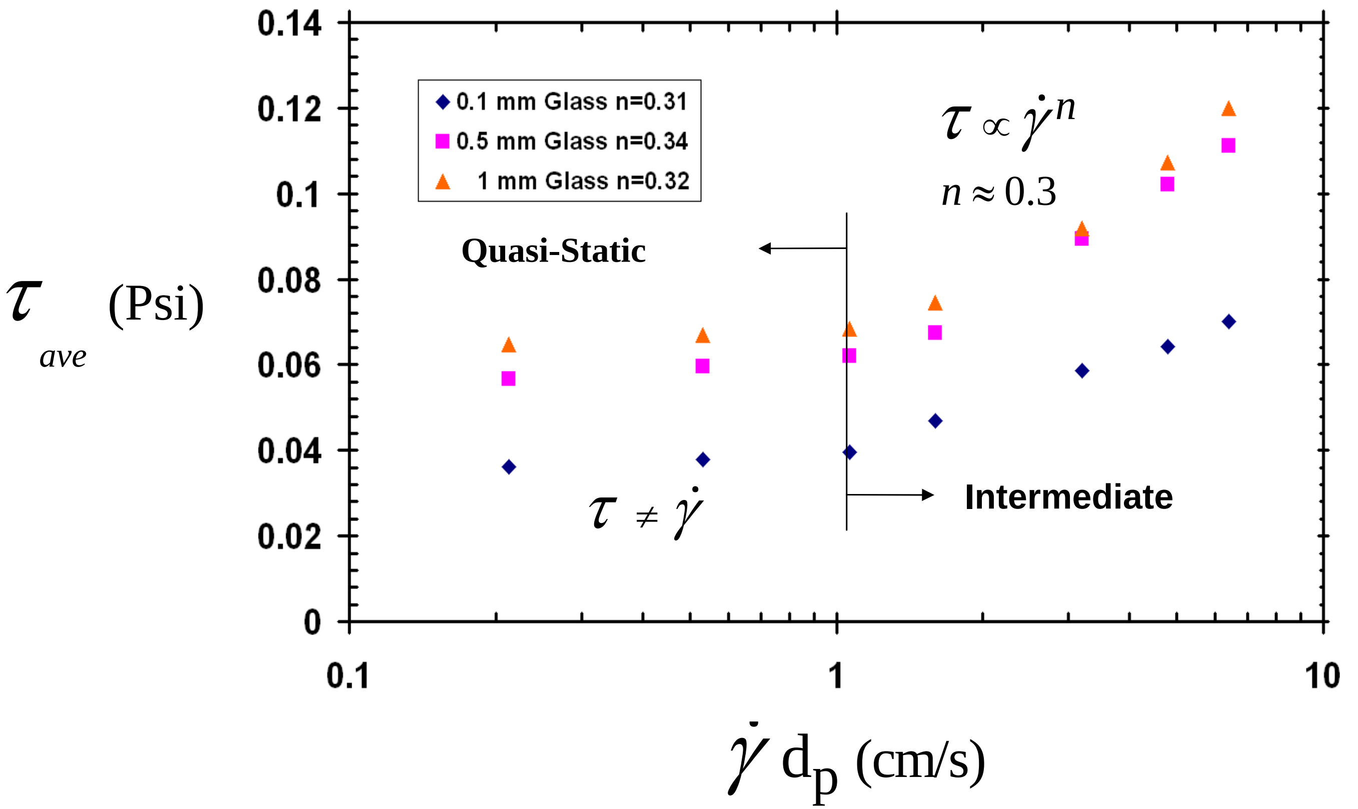


$$\tau_{ave} = \frac{2T}{\pi L D^2}$$

T – torque
L & D – dimensions of cylinder

- Shear stress is independent of shear rate
- Expansion is constrained by overburden in the no-flow system
- Transition to intermediate regime does not occur

Flow regimes in a continuous Couette



- **Continuous Couette**

$$\tau_{\text{ave}} = \frac{2T}{\pi L D^2}$$

T – torque
L & D – dimensions of cylinder

- Bed can dilate and solid concentration decreases
- At low shear rates: the quasi-static regime dominates
- Increasing the shear rate changes the regime of flow to intermediate

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Summary

- DEM simulations of simple shear flows of dense granular materials reveal the different regimes and the appropriate ‘cohesive scaling” for the stresses.
- The apparent coefficient of friction is a strong function of particle volume
- Jenike cell simulations and experiments show qualitative agreements.

Summary - contd

- A stress transition is found in unsteady shear flows following flow reversal and is correlated to the microstructure evolution.
- Boundary layer effects have been identified with the presence of walls, but much more characterization is needed.
- Preliminary experiments have been done using a continuous flow Couette cell, with more results to follow.

Future work

- Refine hypoplastic models-- incorporate fabric evolution (Princeton)
- Simulate unsteady shear in Couette cell (Princeton)
- Perform Couette flow experiments (CCNY)
- Compute order parameter from more DEM simulations (DEM)
- Quantify boundary layer effects and compare to predictions from continuum models (ISU)

Thank you!